A GENERAL-PURPOSE COMPUTER PROGRAM FOR THE VOLTERRA-SERIES ANALYSIS OF NONLINEAR MICROWAVE CIRCUITS

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ABSTRACT

This paper describes a computer program that performs a Volterra-series analysis of a weakly nonlinear microwave circuit having an arbitrary topology. It uses the method of nonlinear currents and a nodal formulation. When special algorithms are used to evaluate only the minimum necessary set of mixing products, the computational efficiency is very great.

INTRODUCTION

Volterra-series, or nonlinear-transfer-function analysis, [1-3], is an efficient and highly effective method for analyzing weakly nonlinear circuits under small-signal excitation. It has been used to analyze nonlinear effects in a wide variety of circuits, and is particularly useful for calculating intermodulation levels in MESFET amplifiers [4,5]. Volterra-series techniques have also been used for the analysis of time-varying circuits such as mixers [6] and parametric upconverters [7]. However, the large amount of algebraic effort necessary to generate analytical forms of the Volterra kernels of even simple circuits has prevented the wide application of Volterra-series analysis. This analytical complexity has forced many experimenters to employ simplifying assumptions, especially in the topology of the equivalent circuits of solid-state devices [4,8]; these approximations limit the accuracy of the analysis.

This paper describes a general-purpose computer program that performs a Volterra-series analysis of a weakly nonlinear microwave circuit. The program is intended primarily for use in the design of microwave circuits; its catalog of circuit elements includes the distributed elements necessary for such work. Because the program formulates and solves the circuit equations numerically, the user need not simplify either the circuit or the model of the solid-state device, or make any of the other common simplifying assumptions often found in Volterra-series analyses (e.g. that the input frequencies in a two-tone excitation are nearly identical).

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THE NONLINEAR-CURRENT METHOD

The program uses the nonlinear-current method [1]-[3], [9] and a nodal formulation of the circuit equations. In this approach, each nonlinear circuit element is described as a linear element in parallel with a set of current sources; each current source represents a single order (greater than 1) of the mixing products, and its current is a nonlinear function of the node-voltage components at lower-order mixing frequencies. The weakly nonlinear circuit is reduced to a linear circuit, which contains the linear elements and the linear parts of the nonlinear elements, and a set of excitation sources.

The first-order response is simply the response of the linear circuit. The source currents at second-order mixing frequencies are found from the first-order node voltages. For example, in the case of a simple voltage-controlled conductance having the small-signal I/V characteristic,

$$i = g_1 v + g_2 v^2 + g_3 v^3 + \dots \qquad (1)$$

where the first-order terminal voltage v(t) is

$$v(t) = \frac{1}{2} \sum_{q=-Q}^{Q} V_{s,q} \exp(j\omega_q t) \qquad (2)$$

the second-order current [3,9] is

$$i_2(t) = g_2 v_1^2(t)$$
$$= \frac{g_2}{4} \sum_{q1=-Q}^{Q} \sum_{q2=-Q}^{Q} V_{1,q1} V_{1,q2} \cdot \exp[j(\omega_{q1}+\omega_{q2})t] \qquad (3)$$

In the case of a capacitor having the charge/voltage characteristic

$$q = C_1 v + C_2 v^2 + C_3 v^3 + \dots. \qquad (4)$$

the second-order current $i_2(t)$ is

$$i_2(t) = C_2 \frac{d[v_1^2(t)]}{dt} \qquad (5)$$

$$= \frac{C_2}{4} \sum_{q1=-Q}^{Q} \sum_{q2=-Q}^{Q} j(\omega_{q1}+\omega_{q2})$$

$$\cdot V_{1,q1} V_{1,q2} \exp[j(\omega_{q1}+\omega_{q2})t]$$

One can find the second-order node voltages by solving the linear network's admittance equations with the second-order sources used as excitations. The process is repeated to find the third- and higher-order node voltages.

Volterra-series analysis is considerably more efficient than generalized harmonic-balance analysis because it requires neither Fourier transformation nor iteration: when the nonlinear-current method is used, the source currents are found directly in the frequency domain. One need not evaluate all the mixing products implied by (3) or (5); it is necessary to evaluate only the small set of products that contribute to the distortion product of interest. Thus, if only the minimum number of significant products are retained, the calculation of the mixing products can be performed very rapidly. Volterra-series analysis is, however, limited to circuits having relatively weak nonlinearities; harmonic-balance techniques can be used with circuits having much stronger nonlinearities.

PROGRAM DESCRIPTION

The program is written in Turbo Pascal for use on the IBM PC or PC-AT microcomputers. The program is menu-driven, and can be used interactively to generate output in several different forms. The element catalog includes the same types of elements found in most linear-analysis programs: linear resistors, capacitors, inductors, controlled sources, transmission lines, stubs, and arbitrary impedances; and nonlinear resistors, capacitors, and controlled sources. At present the program is limited to nonlinear elements that can be defined as voltage-controlled (i.e., whose incremental currents are single-valued functions of voltage). The program has a modular structure, allowing additional types of circuit elements or additional capabilities to be added easily.

Figure 1 shows the program's flow chart. After reading the data file, the program generates the frequency-mix vectors [3] of only the desired and contributory mixing frequencies; i.e., it ignores mixing components of the input tones that do not contribute to the distortion product of interest. The program then finds the source currents and node voltages at each mixing frequency, beginning with the first-order voltages and continuing through the order of the mixing product of interest. At each order, the program first identifies the significant lower-order mixing vectors and the corresponding voltages, calculates the excitation currents, generates the nodal equations, and finally calculates the node voltages.

Figure 2 shows a sample calculation. Figure 2(a) and 2(b) show the circuit and data file; the file describes a single-stage, narrowband, 9.5-GHz GaAs MESFET amplifier. The amplifier's source and load impedances are defined by tabular data in the files ZSFET.DAT and ZLFET.DAT, respectively (the program interpolates these tables if they do not contain data at precisely the correct frequency). The FET equivalent circuit includes the three nonlinear elements that are normally dominant in establishing the device's intermodulation characteristics: the gate/source capacitance, the transconductance, and the drain/source resistance.

The data file lists the excitation frequencies, power levels, and the coefficients of the IM product of interest, followed by the circuit description. The elements *VCN*, *GNL*, and *DIODEC* are nonlinear; *VCN* and *GNL*, the controlled current source and drain/source conductance, are described by the power-series coefficients of their incremental I/V characteristics. *DIODEC*, representing the gate/source capacitance, is the capacitance of an ideal, uniformly doped *pn* junction. The capacitance is described by its junction parameters (C_{j0}, ϕ, and γ) and bias voltage; the program calculates the coefficients of its charge/voltage characteristic from these data.

Figure 2(c) shows the results of a two-tone sweep across the amplifier's passband: it lists output powers of the linear responses at the excitation frequencies f_1 and f_2, and third-order IM products at the $2f_2-f_1$ mixing frequency. The output listing includes the source and load impedances at the input and IM frequencies. Computation speed is limited primarily by the solution of the admittance equations, and is almost independent of the number of nonlinear elements; thus, many more elements could be treated as nonlinear ones without a significant increase in calculation time. The data in Figure 2(c) required only 2½ seconds of computer time per line.

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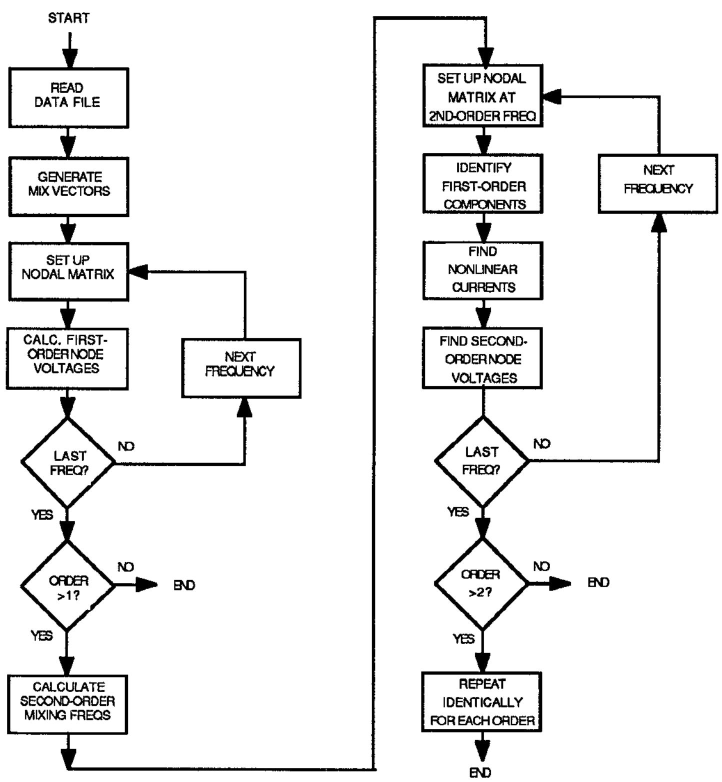
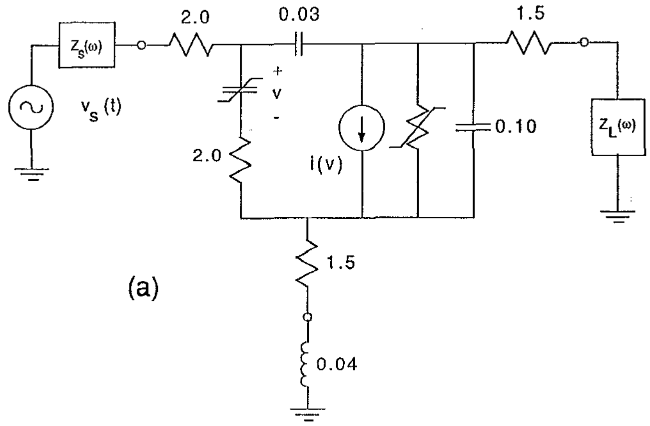


Fig. 1: Program flowchart

Fig. 2: (a) FET amplifier equivalent circuit; (b) data file; (c) output listing.




```
!
!   ***********************************
!   ********** FET IM STUDY **********
!   ***********************************
!
FREQS  5.5E9      5.51E9
dBm    -20.0      -20.0
COEFS  -1          2
!
CKT
 ZS    1  ZSFET.DAT
 ZL    7  ZLFET.DAT
 RES 1 2 2.0
 RES 3 5 2.0
 RES 5 6 1.5
 RES 4 7 1.5
 IND 6 0 0.04E-9
 CAP 2 4 0.030E-12
 CAP 4 5 0.1E-12
 VCN 2 3 4 5 0.03311 -0.00401 -0.00426  0 0
 DIODEC 2 3  0.45e-12    0.7   0.500   -1.5
 GNL  4 5    0.00367 -0.000370 0.000144 0 0
END
```

(b)

VERMIN

Ver. I.1
12/24/87

File Name: fetxx.dat

Initial Input Freqs and powers

Component	Freq.	Power(dBm)
f1	5.500E+009	-20.0000
f2	5.510E+009	-20.0000

(c)

Frequency of IM product: fIM = 2f2 - f1   Order of IM product = 3

INPUT FREQUENCIES					F1 OUTPUT PARAMS					IM FREQ	IM OUTPUT PARAMS		
f1	f2	f3	f4	f5	Re{Zs}	Im{Zs}	Re{ZL}	Im{ZL}	Pout	fIM	Re{ZL}	Im{ZL}	Pout
5.500E+009	5.510E+009	0.00	0.00	0.00	47.22	2.50	49.44	9.72	-12.74	5.520E+009	49.42	10.11	-84.02
6.500E+009	6.510E+009	0.00	0.00	0.00	41.67	7.50	48.33	29.17	-12.37	6.520E+009	48.31	29.56	-83.34
7.500E+009	7.510E+009	0.00	0.00	0.00	36.11	12.50	47.22	48.61	-11.74	7.520E+009	47.20	49.00	-81.24
8.500E+009	8.510E+009	0.00	0.00	0.00	30.56	17.50	46.11	68.06	-10.93	8.520E+009	46.09	68.44	-77.52
9.500E+009	9.510E+009	0.00	0.00	0.00	25.00	22.50	45.00	87.50	-10.19	9.520E+009	45.00	87.50	-72.90
1.050E+010	1.051E+010	0.00	0.00	0.00	15.66	14.06	37.52	72.92	-12.42	1.052E+010	37.35	72.59	-80.93
1.150E+010	1.151E+010	0.00	0.00	0.00	5.29	4.69	29.20	56.71	-18.87	1.152E+010	29.04	56.39	-102.09
1.250E+010	1.251E+010	0.00	0.00	0.00	0.10	0.00	20.89	40.51	-38.30	1.252E+010	20.72	40.19	-160.51
1.350E+010	1.351E+010	0.00	0.00	0.00	0.10	0.00	12.57	24.31	-40.80	1.352E+010	12.41	23.98	-164.53
1.450E+010	1.451E+010	0.00	0.00	0.00	0.10	0.00	4.26	8.10	-46.23	1.452E+010	4.09	7.78	-169.92
1.550E+010	1.551E+010	0.00	0.00	0.00	0.10	0.00	0.10	0.00	-62.95	1.552E+010	0.10	0.00	-186.12